Chettinad College of Engineering & Technology
Approved by AICTE-New Delhi and Affiliated to Anna University-Chennai

**Academic Year 2023 -2024**

**Notes of Lesson**

| | | |
|---|---|---|
| **Year/Semester:** I/II<br>**Date:** 06.03.2024 | **Department** : ECE<br>**Subject Code/Title :** EC3251/ Circuit Analysis<br>**Faculty Name** : Mrs.A.Karthikeyani | **Unit** : III<br>**Total Hours** : 60 Hrs<br>**Subject Credit : 4** |

**Unit III: Sinusoidal Steady State Analysis**

UNIT – III – Sinusoidal Steady State Analysis

Sinusoidal steady state analysis, characteristics of sinusoids, The complex forcing function, The phasor, phasor – relationship for R, L & C, Impedance and Admittance, Nodal and Mesh Analysis, phasor diagrams, AC circuit power Analysis, Instantaneous power, Average power, Apparent power and power factor, complex power

A.C Fundamentals:

Sinusoidal s/l:

⇒ "Fundamental s/l which is used to describe a smooth periodic oscillations" (gives same s/l repeatedly as we are positioned as shown) [Visual ...]

⇒ Mathematically, it can be defined as

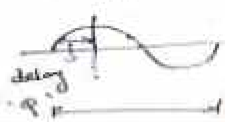
$$\boxed{x(t) = A\sin(2\pi f t + \varphi)}$$

where → $x(t)$ → s/l → time varying function
$A$ → amplitude
$f$ → frequency $(1/T)$
$\varphi$ → phase angle

$\omega = 2\pi f$ ⇒ Angular frequency


[ the same s/l will be repeated ]

⇒ $\varphi$:




$\varphi$ ⇒ may be advanced (or) delay.

Application of sine wave:

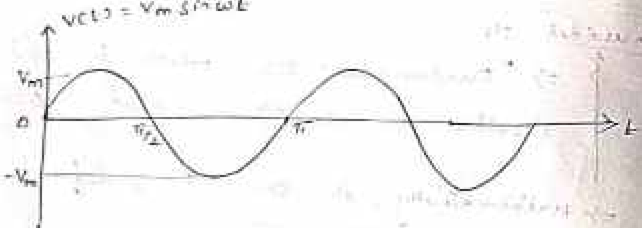
(1) Can be used as simple building blocks to describe any periodic waveform

(2) widely used in the field of mathematics, physics, engineering, etc.,

Examples:

In real time, used in GPS, tracking systems, electrical appliances, cell phones, etc.,

Ex




$\therefore V_{p-p} = 2V_m$

↓

peak to peak voltage

$\omega = 2\pi f = \frac{2\pi}{T}$

$f = \frac{\omega}{2\pi}$

$\therefore \boxed{T = \frac{2\pi}{\omega}}$

Suppose if,

$V_\theta(t) = V_m \sin(\omega t \pm \theta)$

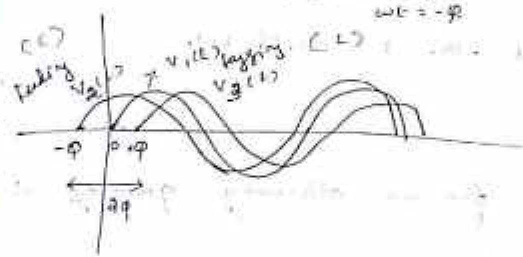
$V_1(t) = V_m \sin(\omega t)$

$V_2(t) = V_m \sin(\omega t + \theta)$

$V_3(t) = V_m \sin(\omega t - \theta)$

$v_2(t) \Rightarrow \omega t + \varphi = 0$

$\omega t = -\varphi$




⇒ To compare 2 sinusoids ⇒ should have same f

" " expression (sine/cos)

" " +ve $V_m$

## Advantages of Sinusoids:

"Note: For arithmetic operations to be performed in AC circuits, it is necessary to find magnitude and angle.

(1) Sinusoidal volt. & I produce less iron and copper losses in AC rotating machines & Transformers. It improves the efficiency.

(2) Sinusoidal V & I will offer less interference (unwanted) to nearby telephone lines

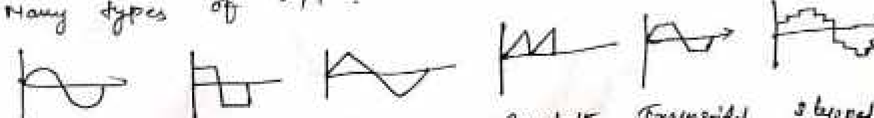
(3) They produce less disturbance in the electrical circuits.

Basic terms:

(i) Waveform: (W/F)

"Graph drawn b/n the alternating quantity as ordinate & time"

Many types of W/F:



Sine, Rectangular, Triangular, Sawtooth, Trapezoidal, Stepped

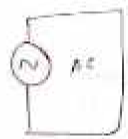
(2) cycle :

"set of +ve and -ve portions of w/f".

(3) Time period :

"Time required for an alternating quantity to complete one cycle

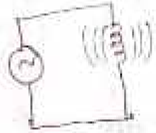
1. Resistance → oppose flow of $e^-$ (or) current


AC → V & I → alternate direction w.r.t freq.

$R = V/I$

Reactance :



⇒ when ct' flow thro' the Inductor → magnetic field will be produced

↓

This will generate energy → supplied to the load.

→ opposes change in current

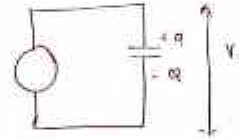
→ Reactance offered by 'L' ⇒ '$X_L$'

$X_L = 2\pi f L$ (Ω)

$f$ = frequency

$L$ = Inductance

$X_L \propto f$




⇒ '$X_C$' ⇒ oppose sudden changes of voltage

$X_C = \frac{1}{2\pi f C}$ (Ω)

$X_C \propto \frac{1}{f}$

(R, L), (R, C), (L, C), (R, L, C)


⇒ Impedance : [Resistance + Reactance]

$Z = \sqrt{R^2 + X_L^2}$ Ω

$V = IZ$, $I = V/Z$ ; $Z = \frac{V}{I}$

Phasor and phase diagram :

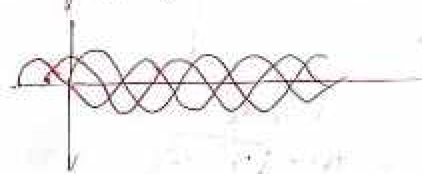
Phasor :

⇒ Representation of a sinusoidal signal in complex number

⇒ In terms of polar form, ⊏⊐r form (or) Exp. vector form

Phase diagram : (Explains the relation b/w I & V)

⇒ particularly useful when we have more than one sinusoidal signal which is having same frequency but diff amplitude & phase

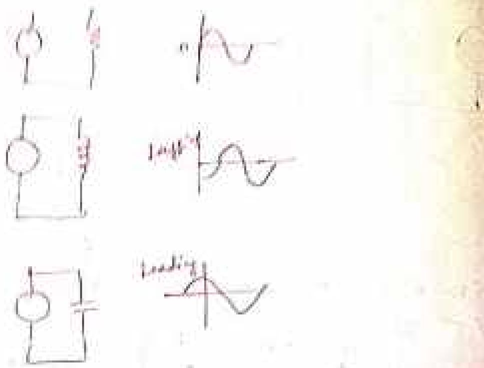


Load type | circuit | V/I w/t | Vector diagram

R

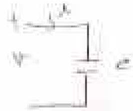
L — lagging

C — leading



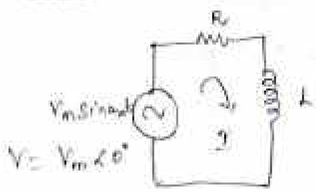
Phasor Relationship for 'L':

$V = (j\omega L) I$



$V = \left(\frac{1}{j\omega C}\right) I$

Series RL circuit phasor diagram:

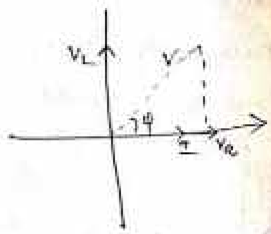



$V = V_m \angle 0^\circ$

Using KVL,

$V = V_R + V_L$

$= IR + I(j\omega L)$




$V = V_R + V_L$

$|V| = \sqrt{V_R^2 + V_L^2}$

$\varphi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$

$V(t) = V_m \sin(\omega t)$

$I(t) = V_m \sin(\omega t - \varphi)$

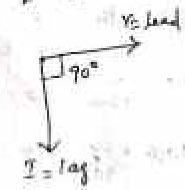
$V_{rms} = \frac{V_m}{\sqrt{2}}$

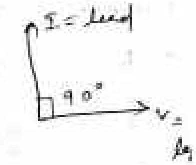
$I_{rms} = \frac{I_m}{\sqrt{2}}$

$\Rightarrow$ phasor diagram direction is anticlockwise

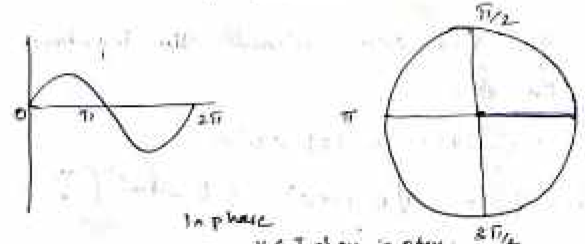
Resistive circuit | 'L' circuit | 'C' circuit


Same phase



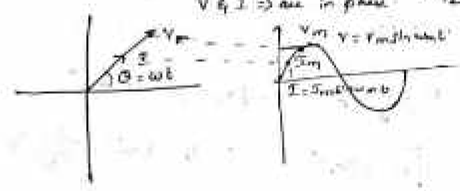

Note: AC circuits → will keep change in magnitude & direction w.r.to time

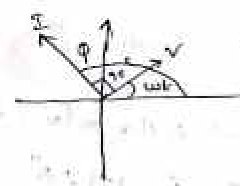



In phase

V & I ⇒ are in phase

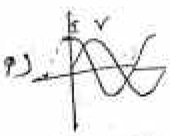
Resistor




$V = V_m \sin \omega t$

$I = I_m \sin(\omega t + \varphi)$

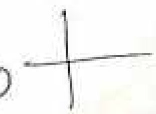




$V = V_m \sin \omega t$

$I = I_m \sin(\omega t - \frac{\pi}{2})$

lag.



## Impedance in A.C circuit

⇒ In A.C circuit along with the R, L, & C also play an important role.

⇒ $X_L = \omega L = 2\pi f L \ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \ \Omega$

| parameter | characteristics | Z in Rectangular | Z in polar | phase diagram |
|---|---|---|---|---|
| Pure R | V & I in phase | $Z = R+j0$ | $Z = R\angle 0^\circ$ |  |
| " L | I lags V by $90^\circ$ | $Z = 0+jX_L$ | $Z = X_L\angle 90^\circ$ |  |
| " C | I leads V by $90^\circ$ | $Z = 0-jX_C$ | $Z = X_C\angle -90^\circ$ |  |

Thus for R-L series circuit, the impedance is represented as,

$Z = R+jX_L = |Z|\angle\theta^\circ \ \Omega$

where ⇒ $|Z| = \sqrt{R^2+(X_L)^2}$ & $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

R-C series,

$Z = R+\frac{1}{j}X_C = R-jX_C = |Z|\angle\theta^\circ$

$|Z| = \sqrt{R^2+(X_C)^2}$ & $\theta = \tan^{-1}\left(-\frac{X_C}{R}\right)$

1. ~~First~~ Let $Z = (10+j15)\,\Omega$. Convert this into polar form.

Soln: Given $Z = (10+j15)\,\Omega$ → Rectangular form.

For polar form ⇒ $|Z|\angle\theta^\circ$

$|Z| = \sqrt{Real^2 + imaginary^2}$
$= \sqrt{10^2+15^2} = \sqrt{100+225} = \sqrt{325} = 18.03\,\Omega$

$\theta = \tan^{-1}\left(\frac{Imaginary}{Real}\right)$

$= \tan^{-1}\left(\frac{15}{10}\right)$

$= 56.31^\circ$

∴ $Z = 18.03\angle 56.31^\circ$

Calculator:
→ Type Real part
↓
$\boxed{Shift}$ → $\boxed{R\to P}$
↓
Enter imaginary part
↓
press = (∵ shows magni
↓
press $x \to y$
↓
shows the angle

(2) Convert $20-j15$ into polar form:

Ans: $Z = 25\angle -36.9^\circ$

polar form to Rectangular form

$Z = |Z|\angle\theta^\circ$ ⇒ ~~$Z = |Z|\cos\phi + j|Z|\sin\phi$~~

$Z = |Z|\cos\phi + j$

$Z = r\cos\phi + jr\sin\phi$
$= |Z|\cos\phi + j|Z|\sin\phi$
$x = r\cos\phi$

$= 10\times\cos(60^\circ) + j(10\times\sin(60^\circ))$

R to P:
$x+jy = z\angle\theta$
where
$z = \sqrt{x^2+y^2}$
$\theta = \tan^{-1}(y/x)$

P to R:
$z\angle\theta = x+jy$
$x = r\cos\theta = z\cos\theta$
$y = r\sin\theta = z\sin\theta$

(3) Convert $Z = 10\angle 60^\circ$ into rectangular form:

$r = 10$ ; $\theta = 60^\circ$
(or)
$|Z| = 10$ ; $\theta = 60^\circ$

| | 0 | 1/4 | 1/2 | 3/4 | 1 |
|---|---|---|---|---|---|
| sin | 0 | 1/2 | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1 |
| cos θ | 1 | $\sqrt{3}/2$ | $1/\sqrt{2}$ | 1/2 | 0 |

⇒ $x = |Z|\cos\theta$
$= 10\times\cos(60) = \cancel{10\times\frac{\sqrt{3}}{3}} = 10\times\frac{1}{2} = 5$

Imaginary part:

$y = r\sin\theta = |z|\sin\theta$

$= 10\sin(60)$

$= 8.66$

$\therefore \boxed{z = 5 + j8.66\,\Omega}$

Calculator:

Enter |z|
↓
press $\boxed{shift}$ → $\boxed{P \to R}$
↓
Enter angle ($\theta$)
↓
$-/+$
↓
$=$ → Active part
↓
$x \to y$ → Reactive part

(2) Convert $K = 30\angle 45^\circ$ into rectangular form

Calc:

$30 \to \boxed{shift} \to \boxed{P \to R} \to 45^\circ \to \boxed{-/+} \to \boxed{=}$
↓
21.21 (Active part) → $\boxed{x \to y}$ → $-21.22$

(3) $z_1 = 10 + j10$, $z_2 = 20 - j30$ are in series. Find the $z_T$ in polar form

$z_T = z_1 + z_2 = 10 + j10 + 20 - j30$

$= 30 - j20$

$\therefore \boxed{z_T = 36.1\angle -33.7^\circ}$

(6) • The total impedance of a circuit in which $z_1$ & $z_2$ are in series is equal to $(30 + j40)\Omega$, $z_1 = (20 + j60)\Omega$. Find $z_2$

$z_T = z_1 + z_2$

$z_2 = z_T - z_1$

$= 30 + j40 - 20 - j60$

$= (10 - j20)\Omega$ Rec. form

$\Rightarrow$ polar form $\Rightarrow z_2 = 22.36\angle -63.43^\circ$

(7) In a given circuit $I = 10\angle 60^\circ$, $z = 20\angle 30^\circ$ Find V

$V = I \cdot z = 10\angle 60^\circ (20\angle 30^\circ)$

$= 200\angle(60 + 30)$

$= 200\angle 90^\circ$

(8) In a circuit $V = 200$ Volts; $I = 10\angle 30^\circ$ Find z both in polar & rectangular form

$I = \frac{V}{z} \Rightarrow z = \frac{V}{I} = \frac{200\angle 0^\circ}{10\angle 30} = 20\angle 0^\circ - 30^\circ$

$= 20\angle -30^\circ$

Problems on Vrms:

(9) Write the polar form of the voltage given by, $v = 100\sin(100\pi t + \pi/6)V$. Obtain it rectangular form.

Given, $V_m = 100$, $\phi = \pi/6$; $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{1.414} = 70.7106\,V$

$\therefore$ In polar form, $70.7106\angle 30^\circ V$

Rectangular form, $61.2371 + j35.3553\,V$.

(10) Two a.c voltages are represented by: $v_1(t) = 30\sin(314t + 45^\circ)$, $v_2(t) = 60\sin(314t + 60^\circ)$. Calculate the resultant voltage $v(t)$ & express in the form of $v(t) = V_m\sin(314t + \phi)$

Given, $V_{m_1} = 30$, $V_{m_2} = 60$

$V_{rms_1} = \frac{30}{\sqrt{2}}$ ; $V_{rms_2} = \frac{60}{\sqrt{2}}$

$V_1 = \frac{30}{\sqrt{2}}\angle 45^\circ$ ; $V_2 = \frac{60}{\sqrt{2}}\angle 60^\circ$

$= 15 + j15\,V$

$V_2 = 21.2132 + j36.7423\,V$

For addition, $\boxed{\phantom{x}}$ coordinates

$V_R = V_1 + V_2$

$= \underline{63.1558 \angle 55^\circ}$

$V_{R(max)} = \sqrt{2} \times V_{rms}$

$= \sqrt{2} \times 63.1558$

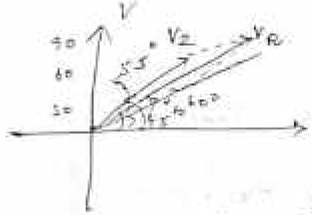
$= 89.3157.$

Note: The voltage will be represented in terms of $V_m$

$V_{rms} = \dfrac{V_m}{\sqrt{2}}$

$\therefore$ The resultant voltage is,

$V(t) = 89.3157 \sin(314t + 55^\circ) V$




(11) Find the impedance of the circuit shown in figure.

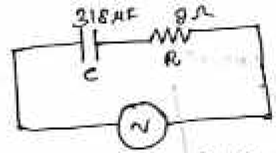

$Z = R + jX_L$

$X_L = \omega L = 2\pi f L = \cancel{2\pi \times 314}\ 314 \times 56.2 \times 10^{-3}$

$= 17.65 \Omega$

$\therefore Z = R + jX_L = 17.68 + j\,17.65 \Omega$

(12) Find the current in the circuit shown in figure and draw the phasor diagram.




Given, $C = 318\mu F$, $R = 8\Omega$, $V_m = 100\sqrt{2}$, $2\pi f = 314\ rad$

$V_{rms} = \dfrac{V_m}{\sqrt{2}} = \dfrac{100\sqrt{2}}{\sqrt{2}} = 100$ & phase $= 0^\circ \Rightarrow$

$V_{rms} = 100 \angle 0^\circ V$

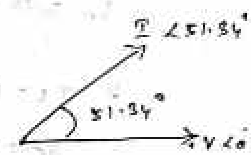
$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{314 \times 318 \times 10^{-6}} = 10\Omega$

$\therefore Z = R - jX_C$

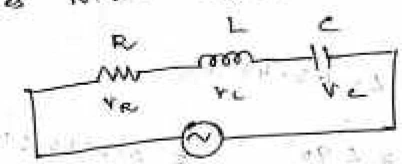
$Z = 8 - j10$

$\therefore I = \dfrac{V}{Z} = \dfrac{100\angle 0^\circ}{12.8062 \angle -51.34^\circ}$

$I = 7.8087 \angle 51.34^\circ$



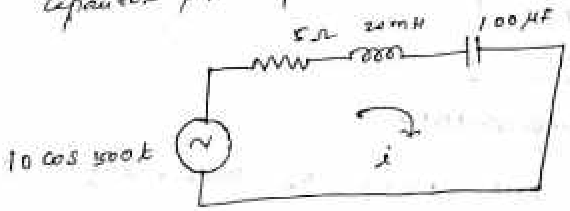

Series R-L-C circuit:




$\therefore Z = R + jX_L - jX_C$

$= R + j(X_L - X_C)$

(13) The network shown in figure is operating in a sinusoidal steady state. Find 'V' across capacitor, R & L.




$V(t) = 10\cos 500t = 10\sin(500t + 90^\circ)$ [$\because \sin(90+\theta) = \cos\theta$]

$R = 5$ $\quad\therefore \theta = 90^\circ$

$L = 20\,mH$

$C = 100\,\mu F$

$\omega = 500 = 2\pi f$

$V_m = 10 \Rightarrow V_{rms} = \frac{10}{\sqrt{2}} \angle 90^\circ = 7.071V \angle 90^\circ$

$X_L = \omega L = 500 \times 20\times10^{-3} = 10\Omega$

$X_C = \frac{1}{\omega C} = 20\Omega$

$\therefore Z_T = 5 + j(10 - 20)$

$= 5 - j10\,\Omega$

$Z_T = 11.1803 \angle -63.43^\circ\,\Omega$

$i = \frac{V}{Z_T} = \frac{7.07\angle 90^\circ}{11.1803\angle -63.43^\circ} = 0.6324\angle 153.43^\circ A$

$\therefore$ ~~$V_R = 0.6324 \angle 153.43 \times 5$~~

$V_R = 0.6324 \times 5 = 3.162V$

$V_L = 0.6324 \times X_L = 0.6324 \times 10 = 6.324V$

$V_C = 0.6324 \times X_C = 0.6324 \times 20 = 12.648V$

Admittance: (A.C parallel circuit)

$Y = \frac{1}{Z}$

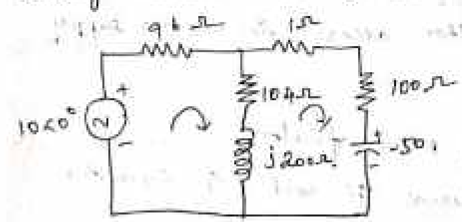
$Y = G + jB$

$G \rightarrow$ conductance

$B \rightarrow$ susceptance

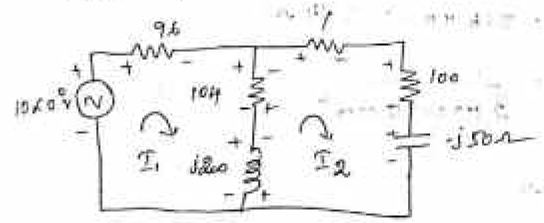
Loop Analysis (or) Mesh Analysis:

(14) Find the current through the source and capacitor in the network shown below, using mesh current analysis.




Step 1:

Show the mesh currents




Step 2: Apply KVL to the two loops.

$-10\angle 0^\circ + 96I_1 + 104I_1 + j200I_1 - 104I_2 - j200I_2 = 0.$

$200I_1 + j200I_1 - (104 + j200)I_2 = 10\angle 0^\circ$

$\Rightarrow (200 + j200)I_1 - (104 + j200)I_2 = 10\angle 0^\circ \rightarrow (1)$

$(205+j150)I_2 - (104+j200)I_1 = 0$

$$\Rightarrow \Delta = \begin{vmatrix} 200+j200 & -104-j200 \\ -104-j200 & 205+j150 \end{vmatrix}$$

$\therefore \Delta = (200+j200)(205+j150) - (-104-j200)(-104-j200)$

$\therefore I_1 = 0.05101 \angle 0^\circ A$

$I_2 = 0.04527 \angle 26.335^\circ A$

Part C - May/June 2022

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(15) A coil of $R = 10\Omega$, $L = 0.1H$ is connected in series with $150\mu F$ capacitor across $200V$, $50Hz$ supply.

Calculate,

(i) $X_L$, $X_C$, $Z$, $I$ & power factor

(ii) The voltage across the coil & capacitor

Solution:

$X_L = 2\pi f \times L = 2\pi \times 50 \times 0.1$

$= 314 \times 0.1 = 3.14\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$

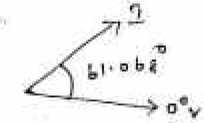
$= 21.23\Omega$

$Z = R + jX_L - jX_C$

$= 10 - j18.091\Omega$

$I = \frac{200}{10 - j18.091} = \frac{200\angle 0^\circ}{20.671\angle -61.068^\circ}$

$\Rightarrow I = 9.675 \angle 61.068^\circ$




$\phi = 61.068^\circ$

$\Rightarrow$ power factor $= \cos\phi$

$= \cos(61.068^\circ)$

$= 0.4838$.

(ii)

$V_L = 9.675 \times 3.14 = 30.3795V$

$V_C = 9.675 \times 21.23 = 205.4003V$

May/June (April/May-2022) (Or)

(16) A resistance of $100\Omega$ is connected in series with a $50\mu F$ capacitor when the supply voltage is $200V$, $50Hz$. Find the

(i) Impedance, current & power factor

(ii) The voltage across the resistor & across capacitor. Draw the phasor diagram.

Solution:

Given:

$R = 100\Omega$, $C = 50\mu F$, $V = 200V = 200\angle 0^\circ$,

$f = 50Hz$.

(i) $Z = R + (-jX_C) = R - jX_C$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f \times C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$

$= 63.69\Omega$

$\therefore \boxed{Z = 100 - j63.69\Omega}$

$$I = \frac{V}{Z_T} = \frac{200\angle 0^\circ}{100 - j63.69}$$

$$|Z| = \sqrt{100^2 + 63.69^2} = 118.56\angle -32.49^\circ$$

$$\therefore \boxed{I = 1.69\angle 32.49^\circ \text{ A}}$$

$V = 200\angle 0^\circ$ ; $I = 1.69\angle 32.49^\circ$

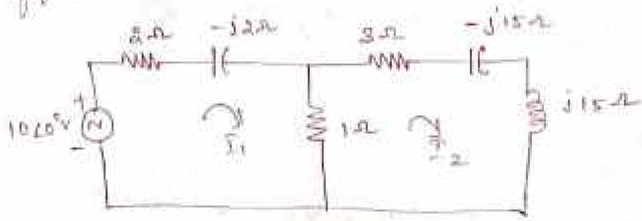
power factor $= \cos\phi = \cos(32.49) = 0.8435$

$V_R = 1.69 \times 100 = 169$ volts

$V_C = 1.69 \times 50\times10^{-6} = 84.5\,\mu V \rightarrow$ (or) $0.0845$ mV

## Loop Analysis (or) Mesh Analysis

(17) Apply mesh current method and determine the currents through the resistors of the network shown in figure



Soln: Apply KVL to loop 1,

$$-10\angle 0^\circ + (3 - j2)I_1 - I_2 = 0$$

$$\Rightarrow (3 - j2)I_1 - I_2 = 10\angle 0^\circ$$

Apply KVL to loop 2,

$$-I_1 + (3 + j0)I_2 = 0$$

$$-I_1 + 3I_2 = 0$$

$\therefore$ In matrix,

$$\begin{bmatrix} 3 - j2 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\Delta = 11 - j8 = 13.6\angle -36^\circ$$

$$\Delta_1 = 40\angle 0^\circ ; \ \Delta_2 = 10\angle 0^\circ$$

$$\therefore I_1 = \frac{\Delta_1}{\Delta} = \frac{40\angle 0^\circ}{13.6\angle -36^\circ} = 2.94\angle 36^\circ \text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{10\angle 0^\circ}{13.6\angle -36^\circ} = 0.735\angle 36^\circ \text{ A}$$

$$(I_1 - I_2) = (2.94\angle 36^\circ - 0.735\angle 36^\circ)$$

$$= 2.379 + j1.728 - 0.595 - j0.432$$

$$= 1.784 + j1.296$$

$$= 2.205\angle 36^\circ$$

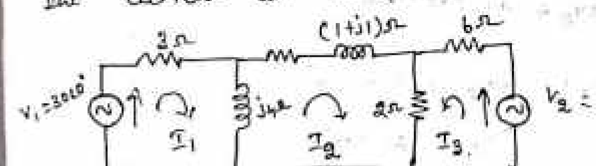
$\therefore I_1 = 2.94\angle 36^\circ$

$I_2 = 0.735\angle 36^\circ$

$I_1 - I_2 = 2.205\angle 36^\circ$

$\therefore$ ~~The current through $2\Omega$ = 3 $10\angle 0^\circ$~~

(18) In the network shown in figure find $V_2$ such that the current in the $(1+j1)\Omega$ branch is zero.



Solution:

The directions of the loop currents are already given in the problem. Without changing the direction, the problem is solved.

Given that the current thru, $(1+j3)\Omega = 0$

$$\therefore I_2 = \frac{\Delta_2}{\Delta} = 0$$

$$\Rightarrow \boxed{\Delta_2 = 0}$$

In the matrix form, we get the loop equation as below:

$$\begin{bmatrix} 3+j4 & -j4 & 0 \\ -j4 & 3+j5 & +2 \\ 0 & 2 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0^\circ \\ 0 \\ V_2 \end{bmatrix}$$

$$\therefore \Delta_2 = \begin{vmatrix} 3+j4 & 30\angle 0^\circ & 0 \\ -j4 & 0 & 2 \\ 0 & V_2 & 8 \end{vmatrix} = 0$$

$$(3+j4)(-2V_2) - 30\angle 0^\circ(-j32) = 0$$

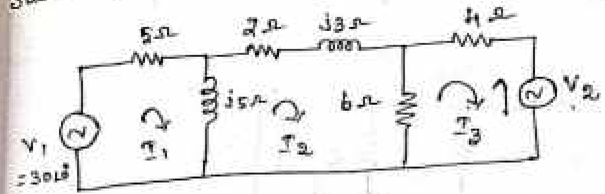
$$-5\angle 53^\circ \times 2V_2 + 960\angle 90^\circ = 0$$

$$+10\angle 53^\circ \times V_2 = +960\angle 90^\circ$$

$$V_2 = \frac{960\angle 90^\circ}{10\angle 53^\circ}$$

$$\boxed{V_2 = 96\angle 37^\circ \text{ volts}}$$

(19) In the network shown in figure determine $V_2$ such that the current in $2+j3$ impedance is zero.




Solution:

In the third loop, the current is in clockwise direction.

Given that, The current thru' $(2+j3)$ is zero.

$$\Downarrow$$

$$I_2 = 0$$

$$\Rightarrow I_2 = \frac{\Delta_2}{\Delta} = 0$$

$$\begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0^\circ \\ 0 \\ -V_2 \end{bmatrix}$$

$$\Delta_2 = \begin{vmatrix} 5+j5 & 30\angle 0^\circ & 0 \\ -j5 & 0 & -6 \\ 0 & -V_2 & 10 \end{vmatrix} = 0$$

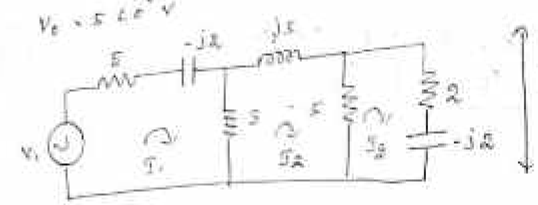
$$\Rightarrow (5+j5)(-6V_2) - 30\angle 0^\circ(-j50) = 0$$

$$(-30-j30)V_2 + 1500j = 0$$

$$V_2 = \frac{+1500j}{+30+j30} = \frac{50j}{1+j1}$$

$$= 25+25j = \underline{35.36\angle 45^\circ}$$

(20) In the n/w shown in figure the source $V_1$ results in a voltage $V_o$ across the $(2-j2)\Omega$ impedance. Find the source $V_1$ which corresponds to $V_o = 5\angle 0^\circ V$




Given, $V_o = 5\angle 0^\circ$.

$$I_3 = \frac{V_o}{2-j2} = \frac{5\angle 0^\circ}{2.83\angle -45^\circ} = 1.77\angle 45^\circ A$$

$$\begin{bmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

$\Delta = 504\angle 4.32^\circ$

$\Delta_3 = V_1(15)$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{V_1(15)}{504\angle 4.32^\circ}$$

$= 1.77\angle 45^\circ$

$V_1 = 59.5\angle 49.32^\circ V$

Nodal Analysis Method:

Node: It is a point where two or more circuit elements terminals are connected together

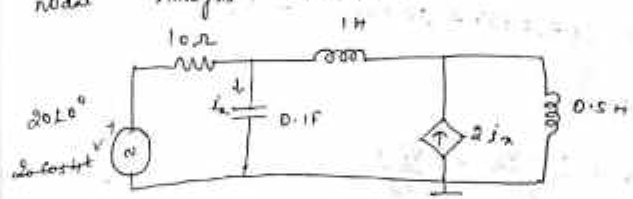
Branch: The connection between node

Steps to Analyze AC circuits

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques
3. Transform the resulting phasor to the time domain

Time to Freq → Solve Variable in Freq → Freq. to Time

(21) Find $i_x$ in the circuit of figure using nodal Analysis. $\omega = 4$ rad.



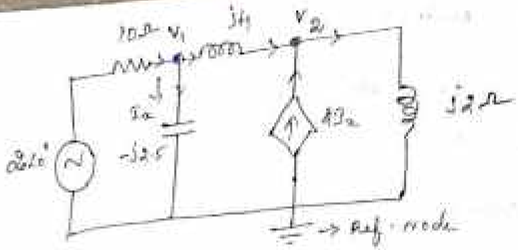

Solution

Given $\omega = 4$

$\Rightarrow$ ~~20 cos 4t~~ $1H = j\omega L = j4$

$0.5H = j4 \times 0.5 = j2$

$0.1F = \frac{1}{j\omega C} = \frac{1}{j(4)(0.1)} = \frac{1}{j(0.4)} = -j2.5$




At Node 1, KCL

$$\frac{20-V_1}{10} = \frac{V_1-0}{-j2.5} + \frac{V_1-V_2}{j4}$$

$$20-V_1 = \frac{10V_1}{-j2.5} + \frac{10(V_1-V_2)}{j4}$$

$$= \frac{4V_1}{-j} + \frac{2.5(V_1-V_2)}{j}$$

$$-j20 \cdot V_1 = 4jV_1 - 2.5j(V_1-V_2)$$

$$20 = V_1 + j4V_1 - 2.5jV_1 + 2.5jV_2$$

$$20 = (1+j2.5)V_1 + j2.5V_2 \longrightarrow (1)$$

At Node 2, KCL,

$$2I_x + \frac{V_1-V_2}{j4} = \frac{V_2-0}{j2}$$

$$I_x = \frac{V_1}{-j2.5} \Rightarrow 2\left(\frac{V_1}{-j2.5}\right) + \frac{V_1-V_2}{j4} = \frac{V_2}{j2}$$

$$j0.8V_1 - j0.25V_1 + j0.25V_2 = -j0.5V_2$$

$$j0.65V_1 + j0.75V_2 = 0 \longrightarrow (2)$$

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ j0.65 & j0.75 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ j0.65 & j0.75 \end{vmatrix}$$

$$= (1+j1.5)(j0.75) - j2.5(j0.65)$$

$$= 0.5 + 0.75j$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & j0.75 \end{vmatrix} = 15$$

$$\therefore V_1 = \frac{15}{0.5+0.75j} = 16.64\angle{-56^\circ}$$

$$V_2 = 13.91\angle 192.3^\circ V$$

$$I_x = \frac{V_1}{j2.5} = 7.59\angle 108.4^\circ A$$

~~Steps~~:

(8) In the network shown in figure, find the node voltages $V_1$ & $V_2$. Find also the current supplied by the source.

$$\frac{20\angle 30 - V_1}{5-j2} = \frac{V_1}{3} + \frac{V_1 - V_2}{j5}$$

$$\frac{20\angle 30}{5-j2} - \frac{V_1}{5-j2} = \frac{V_1}{3} + \frac{V_1 - V_2}{j5}$$

$$\frac{20\angle 30}{5-j2} = \frac{V_1}{5-j2} + \frac{V_1}{3} + \frac{V_1}{j5} - \frac{V_2}{j5}$$

$$V_1\left(\frac{1}{5-j2} + \frac{1}{3} + \frac{1}{j5}\right) - \frac{1}{j5}(V_2) = \frac{8.66+j5}{5-j2}$$

$$V_1\left(\frac{1}{5-j2} + \frac{1}{3} + \frac{1}{j5}\right) - \frac{1}{j5}V_2 = 1.86\angle 51.8^\circ \rightarrow ①$$

Apply KCL at node 2,

$$\frac{V_1 - V_2}{j5} = \frac{V_2}{7-j2}$$

$$0 = -\frac{V_1}{j5} + \frac{V_2}{j5} + \frac{V_2}{7-j2} \rightarrow 0 = -\frac{V_1}{j5} + V_2\left(\frac{1}{j5} + \frac{1}{7-j2}\right)$$

$$0 = -\frac{1}{j5}(V_1) + \left(\frac{1}{7-j2}\right)V_2 \rightarrow ②$$

$$\begin{bmatrix} \frac{1}{5-j2} + \frac{1}{3} + \frac{1}{j5} & -\frac{1}{j5} \\ -\frac{1}{j5} & \frac{1}{7-j2} \end{bmatrix}$$

$$\frac{V_1 - V_2}{j5} = \frac{V_2}{5} + \frac{V_2}{2-j2}$$

$$\frac{V_1}{j5} - \frac{V_2}{j5} = \frac{V_2}{5} + \frac{V_2}{2-j2}$$

$$0 = -\frac{V_1}{j5} + V_2\left(\frac{1}{j5} + \frac{1}{5} + \frac{1}{2-j2}\right) \rightarrow ②$$

$$\begin{bmatrix} \frac{1}{5-j2} + \frac{1}{3} + \frac{1}{j5} & -\frac{1}{j5} \\ -\frac{1}{j5} & \frac{1}{j5} + \frac{1}{5} + \frac{1}{2-j2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.86\angle 51.8^\circ \\ 0 \end{bmatrix}$$

$\Delta =$ ~~(0.51 - j~~

$$\begin{bmatrix} 0.51 - j0.13 & j0.2 \\ j0.2 & 0.45 + j0.05 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.86\angle 51.8^\circ \\ 0 \end{bmatrix}$$

$$\Delta = (0.51 - j0.13)(0.45 + j0.05) - j^2 0.04$$

$$= 0.276 - j0.033 = 0.278\angle -6.82^\circ$$

$$\Delta_1 = \begin{vmatrix} 1.86\angle 51.8^\circ & j0.2 \\ 0 & 0.45 + j0.05 \end{vmatrix}$$

$$= (1.86\angle 51.8^\circ)(0.45 + j0.05)$$

$$= (1.86\angle 51.8^\circ)(0.453\angle 6.3^\circ)$$

$$\boxed{\Delta_1 = 0.843\angle 58.1^\circ}$$

$$\Delta_2 = (-j0.2)(1.86\angle 51.8^\circ)$$

$$= (0.2\angle -90^\circ)(1.86\angle 51.8^\circ)$$

$$= 0.372\angle -38.2^\circ$$

$$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{0.843\angle 58.1^\circ}{0.278\angle -6.82^\circ}$$

$$\boxed{V_1 = 3.03\angle 64.9^\circ \text{ V}}$$

||ly,

$$\boxed{V_2 = 1.34\angle -31.38^\circ \text{ V}}$$

From the give fig:- current supplied by source,

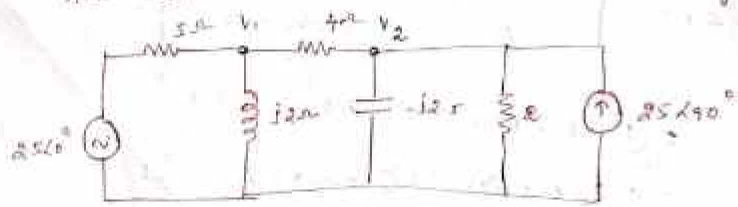
$$\frac{10\angle 30^\circ - V_1}{5-j2} = \frac{10\angle 30^\circ - 3.03\angle 64.9^\circ}{5-j2}$$

$$= \frac{(8.66+j5) - (1.3+j2.75)}{5.39\angle -21.8^\circ}$$

$$= \frac{7.36 - j2.25}{5.39\angle -21.8^\circ} = \frac{7.7\angle -17^\circ}{5.39\angle -21.8^\circ}$$

$$= 1.43\angle 4.8^\circ A$$

(22) Using nodal Analysis, find the current through 4Ω resistor in the circuit shown in figure




Formula:

$$\begin{bmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_B} \\ \frac{-1}{R_B} & \frac{1}{R_B}+\frac{1}{R_C}+\frac{1}{R_D} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_1}{R_A} \\ \frac{V_2}{R_B} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{j2}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{-j2.5}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{25\angle 0^\circ}{5} \\ 25\angle 90^\circ \end{bmatrix}$$

$$\Delta = 0.513\angle -22.3^\circ$$

$$\Delta_1 = 9.06\angle 65.5^\circ$$

$$\Delta_2 = 17.77\angle 39.3^\circ$$

$\therefore V_1 = 17.66\angle 87.8^\circ V = (0.677 + j17.6)V$

$V_2 = 34.64\angle 61.6^\circ V = (16.47 + j30.47)V$

$\therefore$ 'V' across $4\Omega = V_1 - V_2 = V_2 - V_1$

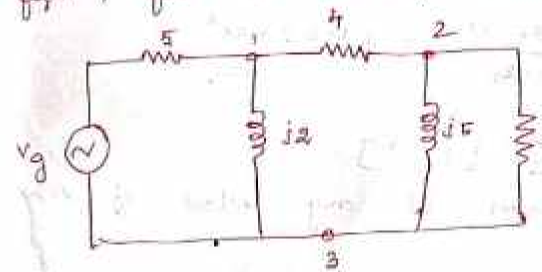
$= 15.793 + j12.87$

$= 20.37\angle 39.17^\circ V$

$I$ thro' $4\Omega = \frac{20.37\angle 39.17}{4}$

$= 5.0925\angle 39.17^\circ A$

(23) Given the nodes 1 & 2 in network of figure, find the ratio of node voltage $V_1/V_2$.




Solution:

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{j2}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{j5}+\frac{1}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_g/5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.45+j0.5 & -0.25 \\ -0.25 & 0.35+j0.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.2V_g \\ 0 \end{bmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0.2V_g & -0.25 \\ 0 & 0.25-j0.2 \end{vmatrix}$$

$$= 0.2V_g(0.25-j0.2)$$

$$\Delta_2 = \begin{vmatrix} 0.45-j0.5 & 0.2V_g \\ -0.25 & 0 \end{vmatrix}$$

$$= 0.2V_g(0.25) = 0.05V_g$$

$$V_1 = \frac{\Delta_1}{\Delta} \; ; \; V_2 = \frac{\Delta_2}{\Delta} \Rightarrow \frac{V_1}{V_2} = \frac{\Delta_1/\Delta}{\Delta_2/\Delta} = \frac{\Delta_1}{\Delta_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{0.2V_g(0.25-j0.2)}{0.05V_g}$$

0.25

$$= \frac{0.25-j0.2}{0.25} = 1.612\angle -30^\circ$$

## Instantaneous Power: [P(t)]

"The electric power at any instant of time"

$\Rightarrow$ watts

$\Rightarrow P(t) =$ Instantaneous 'V' $(v(t)) \times$ Instantaneous 'I' $(i(t))$

$\Downarrow$

By an element

$$P(t) = v(t) \times i(t)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$\therefore P(t) = V_m \cos(\omega t + \theta_v) \times I_m \cos(\omega t + \theta_i)$$

$$= V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\because 2\cos A\cos B = \cos(A-B) + \cos(A+B)$$

$$\therefore = \frac{V_m I_m}{2}\left[\cos(\cancel{\omega t} - \cancel{\omega t} + \theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

Inst. power:

$$\boxed{P(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}_{\text{Time independent term}} + \underbrace{\frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)}_{\text{Time dependent}}}$$

Note: P(t) → dependent on time, difficult to make sure P(t)

## Average power: [Time independent] [$P_{av}$]

"Average of instantaneous power over one period"

$$P_{av} = \frac{1}{T}\int_{t=0}^{t=T} P(t)\,dt$$

$t \rightarrow \omega t$

$t = 0 \Rightarrow 0$

$t = T \Rightarrow \frac{2\pi}{T} \times T = 2\pi$

(or)

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\,dt$$

$$\Rightarrow P_{av} = \frac{1}{T}\int_0^T \left[\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\right] dt$$

$$= \frac{1}{T}\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\int_0^T dt + \underbrace{\frac{1}{T}\times\frac{1}{2}\times V_m I_m\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt}_{0\ [\text{sinusoidal fn over one period}]}$$

$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \left(\frac{1}{T} \times T\right) + 0$

$\Rightarrow P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

(or)

$= \frac{1}{2} V_m I_m \cos(\theta_i - \theta_v)$

$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$

$\boxed{P_{av} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)}$

**Case i** if $\theta_v = \theta_i$ [Both 'V' & 'I' are in same phase]

$P_{av} = V_{rms} I_{rms} \cos(0^\circ)$

$P_{av} = V_{rms} I_{rms}$

(or)

$P_{av} = \frac{1}{2} I_m R \cdot I_m = \frac{1}{2} I_m^2 R$ [in 'R' $\theta_v = \theta_i$]

$\left(\frac{I_m}{\sqrt{2}}\right)^2 = I_{rms}^2$

**Case ii**

if $\theta_v - \theta_i = \pm 90^\circ$ [$\cos 90^\circ$ & $\cos(-90^\circ) = 0$]

$P_{av} = 0 \Rightarrow$ purely reactive circuit

Note: The 'R' will absorb the power all the time & L & C absorb zero Avg. Power.

$\vec{V}$ in phasor $= V_m \angle \theta_v$

$\vec{I}$ " $= I_m \angle \theta_i$

$\vec{I}^* = I_m \angle -\theta_i$

$\vec{V} \vec{I}^* = V_m I_m \angle(\theta_v - \theta_i)$

$= V_m I_m [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)]$

$\therefore Re\{\vec{V} \vec{I}^*\} = V_m I_m \angle(\theta_v - \theta_i)$

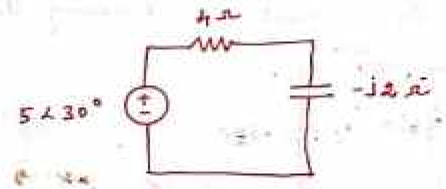
$\vec{V} \vec{I}^* = V_m I_m \angle \theta_v - \theta_i$

$= V_m I_m [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)]$

$\frac{1}{2} Re[\vec{V} \vec{I}^*] = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = P_{av}$

$\vec{V} = V_m \angle \theta_v$, $\vec{I} = I_m \angle \theta_i$

(Q4) For the circuit shown, Find the average power supplied by the source and the average power absorbed by the resistor.




Solution:

$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$V_m = ?$ ; $I_m = ?$ , $\theta_v = ?$ ; $\theta_i = ?$

Given, $\vec{V} = V_m \angle \theta_v$

$\therefore V_m = 5$ ; $\theta_v = 30^\circ$

$Z = (4 - j2)\,\Omega = 4.472 \angle -26.57^\circ$

$\Rightarrow \frac{\vec{V}}{Z} = \frac{5\angle 30^\circ}{4.47\angle -26.57^\circ} = \vec{I}$

$\therefore \vec{I} = 1.118 \angle 56.57^\circ$ A

$I_m = 1.118$ ; $\theta_i = 56.57^\circ$

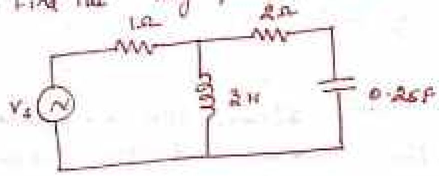
$\therefore P_{av} = \frac{1}{2} \times 5 \times 1.118 \cos(30 - 56.57^\circ)$

$\boxed{P_{av} = 2.5W} \Rightarrow$ By source.

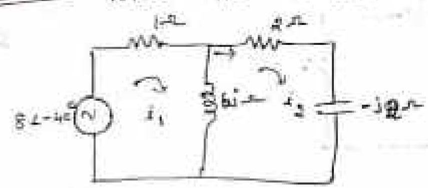
∴ W.K.T, C will not absorbed by the Pav

∴ $\boxed{P_{av}(R) = P_{av}(source) = 2.5\ W}$

(25) Assuming that $V_s = 8\cos(2t-40^\circ)$ V in the below circuit. Find the Avg. power delivered to $2\Omega$ 'R'.




Solution: Draw the circuit in frequency domain




To find Pav, we need find 'i' through '$2\Omega$' ($i_2$)

$$\begin{bmatrix} 1+j6 & -6j \\ -6j & 2+j4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 8\angle -40^\circ \\ 0 \end{bmatrix}$$

$$\Delta_2 = \begin{vmatrix} 1+j6 & 8\angle -40^\circ \\ -6j & 0 \end{vmatrix}$$

$= 6j(8\angle -40^\circ) = 6\angle 90^\circ\,(8\angle -40^\circ)$

$= 48\angle 50^\circ$

$$\Delta = \begin{vmatrix} 1+j6 & -6j \\ -6j & 2+j4 \end{vmatrix}$$

$= (1+j6)(2+j4) + 36$

$= 2 + j4 + j12 - 24 + 36$

$= 14 + j14$

$= 19.78\angle 45^\circ$

38
24
74

$\therefore i_2 = \frac{\Delta_2}{\Delta} = \frac{48\angle 50^\circ}{19.79\angle 45^\circ}$

$\boxed{i_2 = 2.43\angle 5^\circ}$ ⇒ Through '$2\Omega$'

$\therefore P_{av} = \frac{1}{2} \times I_m^2 \times 2$

$= (2.34\angle 5^\circ)^2 = (2.34\angle 5^\circ)(2.34\angle 5^\circ)$

$\boxed{\cancel{P_{av} = 5.48\angle 10^\circ W}}$ $\boxed{P_{av} = (5.06\angle 10^\circ)\,W}$

## Apparent Power and Power factor:

Average power ⇒ $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$P_{av} = \underbrace{V_{rms} I_{rms}}\cos(\theta_v - \theta_i)$

∴ Apparent power (S) $= V_{rms} I_{rms}$

$\therefore P_{av} = S\,\underbrace{\cos(\theta_v - \theta_i)}_{\text{power factor}}$

$= S \times$ power factor

(or) ⇒ power factor (PF) $= \frac{P_{av}}{S}$

$\underbrace{\cos(\theta_v - \theta_i)}_{\text{PF angle}} = \frac{P_{av}}{S}$

If purely 'R', $\therefore \theta_v = \theta_i \Rightarrow \cos 0^\circ = \frac{P_{av}}{S}$

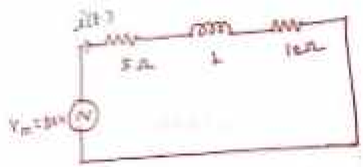
$\Rightarrow \boxed{S = P_{av}}$

If purely reactive load : $\theta_v - \theta_i = \pm 90°$

$$\therefore \cos(\pm 90°) = \frac{P_{av}}{S}$$

$$\Rightarrow 0 = \frac{P_{av}}{S} \Rightarrow \boxed{P_{av} = 0}$$

Q6) In the circuit shown in the given figure, if the "P" consumed by the 5Ω 'R' is 10W, what is the power factor of the circuit.




Solution :

$$PF = \frac{P_{av}}{S} = \frac{P_5 + P_L + P_{10}}{V_{rms} I_{rms}} = \frac{10 + 0 + P_{10}}{50/\sqrt{2} \times I_{rms}}$$

Given, $P_5 = 10$

$I_{rms}^2 \times 5 = 10$ $[\because I_{rms}^2 R = 10]$

$I_{rms}^2 = 2 \Rightarrow I_{rms} = \sqrt{2} = 1.414A = \sqrt{2}A$

$P_{10} = (\sqrt{2})^2 \times 10$

$= 20W$

$$\therefore PF = \frac{10 + 20}{\frac{50}{\sqrt{2}} \times \sqrt{2}} = \frac{30}{50} = 0.6$$

$$\boxed{PF = 0.6}$$

Definition of Apparent power (S):

$$\boxed{S = V_{rms} I_{rms}}$$